UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

0212815821

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2011**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.

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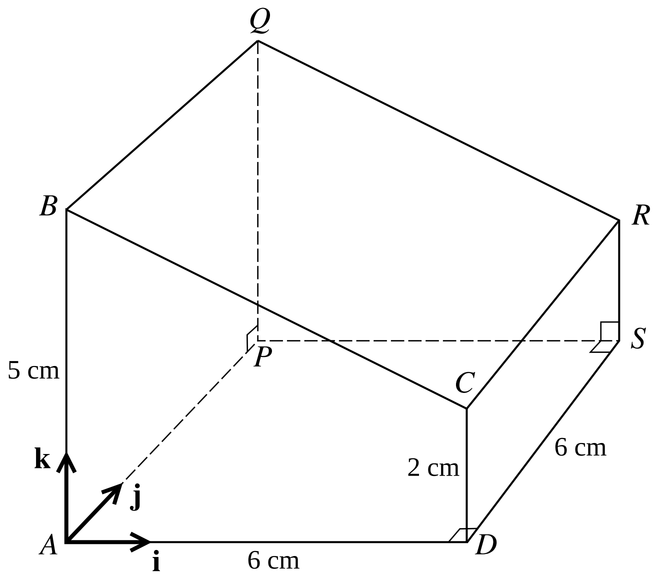
1 Find the coefficient of x in the expansion of $\left(x + \dfrac{2}{x^2}\right)^7$. [3]

2 The volume of a spherical balloon is increasing at a constant rate of 50 cm^3 per second. Find the rate of increase of the radius when the radius is 10 cm. [Volume of a sphere = $\frac{4}{3}\pi r^3$.] [4]

3 **(i)** Sketch the curve $y = (x - 2)^2$. [1]

(ii) The region enclosed by the curve, the x-axis and the y-axis is rotated through 360° about the x-axis. Find the volume obtained, giving your answer in terms of π. [4]

4



The diagram shows a prism $ABCDPQRS$ with a horizontal square base $APSD$ with sides of length 6 cm. The cross-section $ABCD$ is a trapezium and is such that the vertical edges AB and DC are of lengths 5 cm and 2 cm respectively. Unit vectors **i**, **j** and **k** are parallel to AD, AP and AB respectively.

(i) Express each of the vectors $\overrightarrow{CP}$ and $\overrightarrow{CQ}$ in terms of **i**, **j** and **k**. [2]

(ii) Use a scalar product to calculate angle PCQ. [4]

5 **(i)** Show that the equation $2\tan^2\theta\sin^2\theta = 1$ can be written in the form

$$2\sin^4\theta + \sin^2\theta - 1 = 0.$$ [2]

(ii) Hence solve the equation $2\tan^2\theta\sin^2\theta = 1$ for $0° \leqslant \theta \leqslant 360°$. [4]

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6 The variables x, y and z can take only positive values and are such that

$$z = 3x + 2y \quad \text{and} \quad xy = 600.$$

(i) Show that $z = 3x + \frac{1200}{x}$. [1]

(ii) Find the stationary value of z and determine its nature. [6]

7 A curve is such that $\frac{dy}{dx} = \frac{3}{(1+2x)^2}$ and the point $(1, \frac{1}{2})$ lies on the curve.

(i) Find the equation of the curve. [4]

(ii) Find the set of values of x for which the gradient of the curve is less than $\frac{1}{3}$. [3]

8 A television quiz show takes place every day. On day 1 the prize money is \$1000. If this is not won the prize money is increased for day 2. The prize money is increased in a similar way every day until it is won. The television company considered the following two different models for increasing the prize money.

Model 1: Increase the prize money by \$1000 each day.

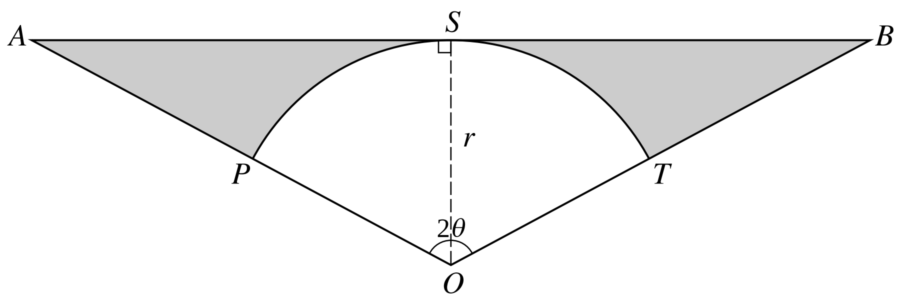
Model 2: Increase the prize money by 10% each day.

On each day that the prize money is not won the television company makes a donation to charity. The amount donated is 5% of the value of the prize on that day. After 40 days the prize money has still not been won. Calculate the total amount donated to charity

(i) if Model 1 is used, [4]

(ii) if Model 2 is used. [3]

9



In the diagram, OAB is an isosceles triangle with $OA = OB$ and angle $AOB = 2\theta$ radians. Arc PST has centre O and radius r, and the line ASB is a tangent to the arc PST at S.

(i) Find the total area of the shaded regions in terms of r and θ. [4]

(ii) In the case where $\theta = \frac{1}{3}\pi$ and $r = 6$, find the total perimeter of the shaded regions, leaving your answer in terms of $\sqrt{3}$ and π. [5]

[Questions 10 and 11 are printed on the next page.]

10 **(i)** Express $2x^2 - 4x + 1$ in the form $a(x + b)^2 + c$ and hence state the coordinates of the minimum point, A, on the curve $y = 2x^2 - 4x + 1$. [4]

The line $x - y + 4 = 0$ intersects the curve $y = 2x^2 - 4x + 1$ at points P and Q. It is given that the coordinates of P are $(3, 7)$.

(ii) Find the coordinates of Q. [3]

(iii) Find the equation of the line joining Q to the mid-point of AP. [3]

11 Functions f and g are defined for $x \in \mathbb{R}$ by

$$\mathrm{f} : x \mapsto 2x + 1,$$
$$\mathrm{g} : x \mapsto x^2 - 2.$$

(i) Find and simplify expressions for $\mathrm{fg}(x)$ and $\mathrm{gf}(x)$. [2]

(ii) Hence find the value of a for which $\mathrm{fg}(a) = \mathrm{gf}(a)$. [3]

(iii) Find the value of b $(b \neq a)$ for which $\mathrm{g}(b) = b$. [2]

(iv) Find and simplify an expression for $\mathrm{f}^{-1}\mathrm{g}(x)$. [2]

The function h is defined by

$$\mathrm{h} : x \mapsto x^2 - 2, \quad \text{for } x \leqslant 0.$$

(v) Find an expression for $\mathrm{h}^{-1}(x)$. [2]

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